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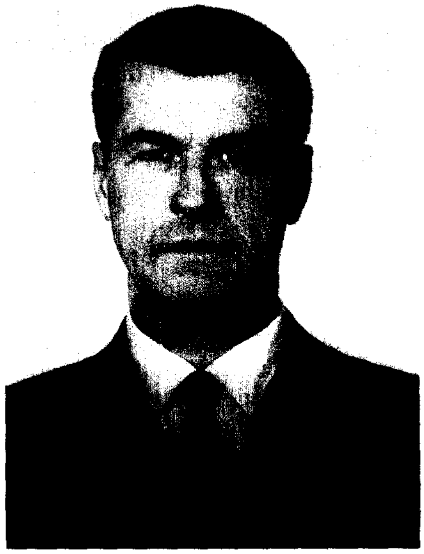

LEV VASIL'YEVICH OVSYANNIKOV (On the occasion of his 80th birthday)†

V. M. TITOV and A. P. CHUPAKHIN

The Russian scientist, Academician Lev Vasil'yevich Ovsyannikov, who has made a major contribution to the development of mechanics and applied mathematics, was 80 years old on 22 April 1999. His classical results in the dynamics of a fluid with free boundaries and transonic gas dynamics are fundamental and the foundation of scientific theories which are being actively developed. Group analysis of differential equations, a fundamental scientific trend at the meeting point of mathematics and mechanics, in which he obtained a number of basic results, is today a powerful and universal instrument for investigating mathematical models in mechanics and physics. His scientific school is well known in world science.

A detailed description of his scientific work can be obtained from the publications associated with his 70th birthday [1, 2]. In this paper, we therefore wish to give a more detailed account of some notable events in his life and scientific activity in the 1990s.

Lev Vasil'yevich Ovsyannikov was born in the town of Vasil'sursk on the Volga. In 1937, he entered the mechanics–mathematics faculty of Moscow University but his studies were interrupted by the war. In the first months of the war, he, together with other students at Moscow State University, helped to construct the defences around Moscow. In the autumn of 1941, he finished his university studies and became a student at the Leningrad Air Force Engineering Academy. After the end of the war in 1945, he became an advanced student at the Academy and subsequently, up to 1953, lectured there and at Leningrad University. His outstanding scientific results in the theory of transonic gas flows date from this period and his candidate dissertation, which he defended in 1949, dealt with these results. We shall mention just a few of them. He discovered for the first time and investigated in detail the special features of gas flow at a straight sonic line and developed a method for analysing such flows. A result which is remarkable in its beauty and importance is due to him: when there is an outflow of a subsonic gas jet with a critical velocity at a boundary, the flow becomes uniform at a finite distance from the aperture. This fact, which became known in transonic gas dynamics as Ovsyannikov's theorem, was one of the first results of a scientific method which was formulated later, namely, the localization of the solutions of degenerate non-linear differential equations.

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At the same time he became familiar with Lie's classical papers on the theory of continuous transformation groups. It is notable that his interest in this theme was initiated by a footnote in the fundamental treatise by Darboux *Leçons sur la théorie générale des surfaces* which he had studied while an advanced student. In this footnote, with a reference to Lie's work, a group was mentioned which is permitted by a differential equation. This combination of algebra and analysis attracted his attention. A second stone in the foundation of a new scientific route was thus laid in the studies of gas dynamics. Up to this time, a large set of exact solutions had been accumulated in gas dynamics but the search for such solutions was based on intuition and subjective considerations of the "simplicity" of the solution.

At the end of the 1940s to the beginning of the 1950s, interest in gas dynamics was to a large extent initiated by problems in aerodynamics and the physics of explosions. He had an excellent knowledge of gas dynamics and a deep understanding of the essence of gas-dynamic phenomena, but his interest in gas dynamics as not solely academic. His contribution to strengthening the nuclear defences of our homeland is significant. In 1953–1956, he worked in the research group directed by Academician M. A. Lavrent'yev [3]. The group considered the problem of developing a nuclear charge for an artillery shell. The dimensional and strength requirements which were imposed on such a shell created fundamental difficulties, not solely of a technological nature. It is interesting that the solutions of the problem of the compression of the charge and of problems in gas dynamics entrusted to him and his colleague S. V. Iordanskii were obtained by calculations on the first large nationally built computer, the BESM-1, which had been developed under the direction of Academician S. A. Lebedev at the Institute of Precision Mechanics and Computational Techniques. The development of the first nationally built nuclear shell for artillery, known as the PDS-41, was a great scientific and technical achievement and an important contribution to the defence capability of the country. Ovsyannikov was awarded the Lenin Prize in 1958 as a member of a group of five leading researchers. The collaboration between Ovsyannikov and Academician M. A. Lavrent'yev subsequently continued at the Moscow Institute of Physics and Technology when it was set up and, later, in Novosibirsk at the time of the organization and establishment of the Siberian Branch of the Academy of Sciences of the USSR and the M. A. Lavrent'yev Institute of Hydrodynamics, where Ovsyannikov was director from 1976 to 1986.

At the Institute of Hydrodynamics, Ovsyannikov continued with his investigations on the group analysis of differential equations which he had started earlier. In 1961, he had defended a dissertation for the degree of Doctor of Sciences in Physics and Mathematics. The theory which he developed and which enabled one to make use of symmetry properties when constructing exact solutions of differential equations, is described in his monograph [4]. This basic work was published in the USA in 1982. For this cycle of investigations on the group analysis of differential equations, he was awarded the State Prize of the USSR (together with N. Kh. Ibragimov) in 1987.

From the beginning of the 1990s, he was intensively and fruitfully occupied with implementing the unique "PODMODELI" scientific program. The concept behind this program and the first results of work carried out under it as applied to a gas dynamic model were published in *Prikladnaya Matematika i Mekhanika* [6]. The principal and most significant progress in the work of the group, which has been directed by him for about six years, is described in a paper published in this issue of the journal [7]. A general group-theory approach to mathematical models is implemented in order to exhaust all possibilities for the exact simplification of large mathematical models by making maximum use of the symmetry properties imposed on them. Such a simplification, the basis of which is group-therapy methods, is achieved by changing to submodels which describe classes of exact particular solutions of the initial model. As a result, a complete atlas, that is, a list of submodels for use in solving physical problems possessing a certain symmetry, is drawn up.

Actually, all the exact solutions of the equations of gas dynamics known today have a group-theory form. The results of work using the PODMODELI program show that they only constitute a small part, the visible tip of an iceberg. The collections of exact solutions contain *thousands* of representatives and, moreover, this number is fairly conditional since many of them still depend on arbitrary functions, that increases their number many times.

In constructing invariant and partially invariant solutions (submodels), it is important to find the substantially different solutions among them, where one cannot be transformed into another by some reversible replacement of the variables. It is found that classes of similar subalgebras of an algebra of invariance which, generally speaking, depend on a certain number of real parameters, correspond to different solutions. The problem of enumerating the different submodels therefore reduces to constructing a set of representatives of the classes of similar subalgebras of the maximal algebra of the invariance of a large model. This set is called the optimal system of subalgebras.

The large volume of work which is required to do this is illustrated, for example, by the optimal system of subalgebras of the algebra of the invariance of the equations of gas dynamics with a general equation of state which Ovsyannikov constructed [6]. This algebra is a Galilean algebra, expanded with a uniform extension operator, which is typical of models of continuum mechanics. The list which has been composed contains more than 200 representatives, and several pages of calculations were required to obtain each of these. Then, hot on the heels of this, he gives a special course "Optimal systems of subalgebras of Lie algebras" at Novosibirsk University, describing the results which have been obtained and refining up to the end the algorithm for constructing optimal systems. This manner of working is characteristic of him: by his own investigations he marks reference points for his own students and researchers. So, the optimal systems of subalgebras for extensions of the basic Galilean algebra in the case of a polytropic gas contain thousands of representatives and were put together by his students on the basis which he had developed, but now using computers [8, 9]. We emphasize that each of these representatives is a potential source of exact solutions of the equations of gas dynamics.

It is noteworthy that, among these, there are examples of physically interesting solutions which it is impossible to obtain from intuitive symmetry considerations. Examples of this are partially invariant solutions, the general theory of which is described in [5]. They generalize classical invariant solutions and differ from them in the fact that, in their case, only a part of the required functions has an invariant representation while the remaining parts, the so-called "superfluous" functions, depend on all the independent variables and satisfy an overdetermined system of differential equations. The reduction of this system to an involution, that is, finding the compatibility conditions, usually involves considerable difficulties. A solution of the singular vortex type [10] is an example of a partially invariant solution with respect to a classical group of rotations. It is characterized by the fact that, in it, the radial component of the velocity is spherically symmetrical but the velocity component which is tangential to the spheres is non-zero. The "superfluous" function is the angle which is formed by the projection of the velocity vector onto the sphere with its meridians. The solution, which is called a singular vortex, is distinguished, apart from this, by the special initial data which ensure the uniqueness and definiteness of the solution on the whole sphere. As an example, a solution is obtained which describes the motion of a gas, initially occupying a spherical layer of finite thickness, which is compressed in the torus-shaped body and collapses to an annulus in the equatorial plane at a finite instant of time.

It is interesting to note that well-known Canadian mathematicians, having considered individual particular solutions of the equations of gas dynamics, were unable to cope with the compatibility analysis of the overdetermined system of equations which describes a singular vortex. Ovsyannikov very elegantly overcame the difficulties in reducing the system to an involution. He obtained exact solutions both for a gas as well as for an ideal incompressible liquid and described their behaviour. This problem clearly illustrates the high level of his mathematical investigations and his profound understanding of the physical content of the solution.

A partially invariant submodel which describes the so-called barochronous gas motions in which the pressure depends solely on time is of interest [11, 12]. One-dimensional solutions of such a form are known in classical gas dynamics. However, new effects are revealed in them when investigating multidimensional motions. In barochronous motions, the trajectories of the gas particles are straight lines. A collapse of the density (it becomes infinite) at a finite instant of time in a manifold of reduced dimensionality is typical of these. The characteristic conoid, which is responsible for the propagation of sonic perturbations and which is constructed on the barochronous solutions, has a singularity close to the collapse manifold: the conoid collapses with respect to this manifold in the case of easily compressible gases.

Ovsyannikov discovered [13] an interesting class of invariant solutions which describe the steady-state two-dimensional flows of a polytropic gas with closed streamlines possessing a discrete symmetry in the case of rotation about the origin of the coordinate system.

Academician Ovsyannikov also established a high scientific level in informal groups of researchers whom he organized to solve urgent scientific problems. For instance, the group monograph [14], which was assembled by pure mathematicians, specialists in mechanics and experimentalists, serves as an example of the complex approach to the description of such a complicated phenomenon as non-linear wave processes. This book presents a wide spectrum of results on wave motion from proofs of correctness and the rigorous establishment of exact mathematical models, the construction of approximate models and the mathematical estimation of their adequacy to their experimental verification. Solitary waves of the elevation and depression type in a two-layer liquid and waves of the smooth step (bora) type, which are solutions of the equations of the second approximation in shallow-water theory, were investigated analytically and obtained experimentally. His cycle of papers "Non-linear theory of the unsteady motions of an ideal liquid with a free boundary", in which a new way of investigating the motion of an ideal liquid in an exact non-linear formulation was described, was awarded the M. A. Lavrent'yev Gold Medal and Prize in 1989. In these papers, an appropriate mathematical apparatus was constructed in which the key element is a theorem on the existence and uniqueness of the solution of the Cauchy problem for a quasidifferential operator in scales of Banach spaces. As applications, the correctness of the Cauchy–Poisson problem in an exact non-linear formulation was proved and the basis of new approximate wave theories, namely linear wave theory and shallow-water wave theory were given.

His extensive arsenal of mathematical methods and his fine understanding of the physics of the phenomena—he generously shares all of this with students and colleagues. The most important thing which he teaches is the ability to free a problem from everything which is superfluous, foreign and extraneous to it ("to remove the garbage" as he graphically expressed himself in an address at one of the conferences). This takes place at a daily discourse, at an actively working seminar and at scientific conferences where he not only lectures but where his rejoinders and brilliant commentaries stick in the minds of the audience. His striving to uncover the essence of a problem and to understand the core of its complexity is an important feature of his scientific and pedagogical work, organizational activity and everyday life. His selfless dedication to science, his true patriotism, which manifests itself in actual deeds and not just in loud words, the high demands which he makes on himself and his constant readiness to help a colleague or a student are inherent traits of this learned Russian patriot.

Academician Ovsyannikov's numerous students, disciples and colleagues, the editorial board and editors of this journal heartily congratulate him on the occasion of this birthday and wish him robust health and the joy of new creative successes.

REFERENCES

1. Lev Vasil'yevich Ovsyannikov (On the occasion of his seventieth birthday). *Prikl. Mat. Mekh.*, 1989, **53**, 2, 179–183.
2. On the seventieth birthday of L. V. Ovsyannikov. *Zh. Prikl. Mekh. Tekh. Fiz.*, 1989, 2, 3–4.
3. NEGIN, Ye. A., GOLEUSOVA, L. P., KULICHOV, G. D. *et al. The Soviet Atomic Project.* Izd. Nizhnii Novgorod, Nizhnii Novgorod, Arzamas, 16, 1995.
4. OVSYANNIKOV. L. V., *Group Properties of Differential Equations.* Izd. Sib. Otd. Akad. Nauk SSSR, Novosibirsk, 1962.
5. OVSYANNIKOV. L. V., *Group Analysis of Differential Equations.* Nauka, Moscow, 1978.
6. OVSYANNIKOV. L. V., The PODMODELI program. Gas dynamics. *Prikl. Mat. Mekh.*, 1994, **58**, 4, 30–55, 1994.
7. OVSYANNIKOV. L. V., Some results of the implementation of the PODMODELI program for the gas dynamics equations. *Prikl. Mat. Mekh.*, 1999, **63**, 3, 362–372.
8. GOLOVIN, S. V., *The optimal system of subalgebras for a Lie algebra of operators admitted by the gas dynamics equations in the case of a polytropic gas.* Preprint No. 5–96, Inst. Gidrodinamiki, Sib. Otd. Ross. Akad. Nauk, Novosibirsk, 1996.
9. CHEREVKO, A. A., The optimal system of subalgebras for the Lie algebra of the operators admitted by a system of gas dynamics equations with an equation of state $p = f(S)\rho^{5/3}$. Preprint No. 4–96, Inst. Gidrodimaniki, Sib. Otd. Ross. Akad. Nauk, Novosibirsk, 1996.
10. OVSYANNIKOV. L. V., The singular vortex. *Zh. Prikl. Mekh. Tekh. Fiz.*, 1995, **36**, 3, 45–52.
11. OVSYANNIKOV. L. V. and CHUPAKHIN, A. P., Regular partially invariant submodels of the gas dynamics equations. *Prikl. Mat. Mekh.* 1996, **60**, 6, 990–999.
12. CHUPAKHIN, A. P., Barochronous gas motions. General properties and submodels of types (1, 2) and (1, 1). Preprint No. 4–98, Inst. Gidrodinamiki, Sib. Otd. Ross. Akad. Nauk, Novosibirsk, 1998.
13. OVSYANNIKOV. L. V., Plane flows of a gas with closed streamlines. *Dokl. Ross. Akad. Nauk*, 1998, **361**, 1, 51–53.
14. OVSYANNIKOV. L. V., MAKARENKO, N. I., NALIMOV, V. I. *et al., Non-linear Problems in the Theory of Surface and Internal Waves.* Nauka, Novosibirsk, 1985.

LIST OF PUBLICATIONS BY L. V. OVSYANNIKOV†

1949

Shock waves on the surface of a profile in a subsonic supercritical flow. *Trudy Leningr. Krasnozn. Voyenno-Vozd. Akad. im A. F. Mozhaiskogo*, 27, 53–54.

A gas flow with a straight transition line. *Prikl. Mat. Mekh.*, **13**, 5, 537–542.

1950

Investigation of gas flows with a straight sonic line. *Trudy Leningr. Krasnozn. Voyenno-Vozd. Akad. im. A. F. Mozhaiskogo*, 33, 25–51.

1952

The equations of the transonic gas motion. *Vestnik Leningrad. Gos. Univ., Ser. Mat., Fiz. i Khim.*, 6, 47–54.

†Articles by L. V. Ovsyannikov of a publicity or popular scientific nature, etc. are not included. *Prikl. Mat. Mekh.* Vol. 63, No. 3, pp. 360–361, 1999.

1953

Tricomi's problem in a class of generalized solutions of the Euler–Darboux equation. *Dokl. Akad. Nauk SSSR*, **91,** 3, 457–460.

1955

The linearization of equations with second-order partial derivatives. *Dokl. Akad. Nauk SSSR*, **102,** 2, 219–221.

1956

General solution of the equations of the renormalization group. *Dokl. Akad. Nauk SSSR*, **109,** 6, 1112–1114.

New solution of the equations of hydrodynamics. *Dokl. Akad. Nauk SSSR*, **111,** 1, 47–49.

1958

Groups and invariant-group solutions of differential equations. *Dokl. Akad. Nauk SSSR*, **118,** 3, 439–442.

1959

Group properties of the equation of non-linear heat conduction. *Dokl. Akad. Nauk SSSR*, **125,** 3, 492–495.

Review: K. G. Guderley, *Theorie schallnacher Strömungen*. Springer, Berlin, 1957. *Novyye Knigi Za Rubezhom, Ser. A*, 8, 25–26.

1960

On the search for the group of a linear second-order differential equation. *Dokl. Akad. Nauk*, **132,** 1, 44–47.

Problems in Elementary Mathematics. Fizmatgiz, Moscow (together with V B. Lidskii, A. N. Tulaikov and M. I. Shabunin).

Group properties of some differential equations of mechanics. In *All-Union Conference on Theoretical and Applied Mechanics*, Moscow, 1960. Book of Abstracts. Moscow, p. 98.

Group properties of S. A. Chaplygin's equation. *Zh. Prikl. Mekh. Tekh. Fiz.*, 3, 126–145.

A note on the calculation of the trace of a non-negative completely continuous operator. *Sib. Mat. Zh.*, **1,** 2, 242–247.

Editor: G. K. Guderley, *Theory of Transonic Flows*. IIL, Moscow.

Editor's Foreword in the above book, pp. 5–6.

1961

Translator: L. Bers, *Mathematical Aspects of Subsonic and Transonic Gas Dynamics*. IIL, Moscow.

1962

Problems in Elementary Mathematics, Second Edition. Fizmatgiz, Moscow (together with V. B. Lidskii, A. N. Tulaikov and M. I. Shabunin).

Application of the theory of Lie groups to certain partial differential equations. Short Communs. Intern. Congr. of Mathematicians, Stockholm: ICM. pp. 103–104.

Group Properties of Differential Equations, Izd. Sib. Otd. Akad. Nauk SSSR, Novosibirsk.

Editor: E. Martensen and K. von Sengbusch, *Calculation of the Transonic Part of Plane and Axially Symmetric Nozzles with a Curvilinear Transition Line*. Izd. Sib. Otd. Akad. Nauk SSSR.

On the convergence of a Mayer series in the case of an axially symmetric nozzle. In the above book, pp. 41–43.

1963

Problems in Elementary Mathematics, Third Edition. Fizmatgiz, Moscow (together with V. B. Lidskii, A. N. Tulaikov and M. I. Shabunin).

On infinite groups of mappings specified by differential equations. *Dokl. Akad. Nauk SSSR*, **148,** 1, 36–39.

Soviet-American Symposium of Mathematicians in Novisibirsk, *Vestn. Akad. Nauk SSSR*, 11, 93–96 (together with T. I. Zelenyak).

Editor and Translator: J. M Serrin, *Mathematical Principles of Classical Fluid Mechanics*. IIL, Moscow.

1964

Soviet-American Symposium on Partial Differential Equations (Novosibirsk, 1963). *Uspekhi Mat. Nauk*, **19,** 2, 241–250 (together with I. I. Danilyuk).

Group properties of the hydrodynamics equations. In *Atto Convegno Lagrangiano (Torino, 1963), Atti Accad. Sci., Torino*, **98,** 357–362.

Partly invariant solutions of the equations admitting a group. In *11th Intern. Congr. on Applied Mechanics*, Munich. Book of Abstracts, S. 1, p. 55.

Invariant-group solutions of the hydrodynamics equations. In *Second All-Union Congress on Theoretical and Applied Mechanics*. Moscow, 1964; Book of Abstracts. Izd. Akad. Nauk SSSR, Moscow, pp. 163–164.

1965

Problems in Elementary Mathematics. Fourth Edition. Fizmatgiz, Moscow (together with V. B. Lidskii, A. N. Tulaikov and M. I. Shabunin).

On a class of unsteady motions of an incompressible fluid. In *Proceedings of the Fifth Session of the Academic Council on the National Economic Utilization of an Explosion*. Ilim, Frunze, pp. 34–42.

Invariant-group solutions of the hydrodynamics equations. In *Proceedings of the Second All-Union Congress on Theoretical and Applied Mechanics*. Moscow, 1964. Nauka, Moscow, No. 2, pp. 302–305.

Singular operator in the scale of Banach spaces. *Dokl. Akad. Nauk SSSR*, **163,** 4, 819–822.

1966

Lectures on the Theory of Group Properties of Differential Equations. Izd. Novosibirsk. Univ., Novosibirsk.

On the theory of a non-local Cauchy problem. In *Proceedings of the International Congress of Mathematicians,* Moscow, 1966. Section 7, p. 48.

Partly invariant solutions of the equations admitting a group. In *Applied Mechanics: Proceedings of the 11th International Congress on Applied Mechanics,* Munich, 1964. Springer, Berlin, pp. 868–870.

1967

Problems in Elementary Mathematics, Fifth Edition. Nauka, Moscow (together with V B. Lidskii, A. N. Tulaikov and M. I. Shabunin).

The Equations of Dynamic Sea Convection. Inst. Gidrodinamiki, Sib. Otd. Akad. Nauk SSSR, Novosibirsk.

Lectures on the Principles of Gas Dynamics. Izd. Novosibirsk Univ., Novosibirsk.

Foreword. Problem on the unsteady motion of a liquid with a free boundary. In *Collection of Scientific Papers of the Theoretical Division*. Nauka, Novosibirsk, pp. 3–4.

General Equations and Examples, ibid., pp. 5–75.

On the "direct" problem of the calculation of a Laval nozzle: Public address at a theoretical seminar at the Institute of Hydrodynamics of the Siberian Branch of the Academy of Sciences of the USSR. *Izv. Akad. Nauk SSSR. MZhG*, 3, 184.

About motion of a finite mass of liquid. In *Fluid Dynamics Trans*., Vol. 3. PWN, Warsaw, pp. 75–81.

Editor: N. Kh. Ibragimov, *Group Properties of Some Differential Equations*. Nauka, Novosibirsk.

The Problem on the Unsteady Motion of a Liquid with a Free Boundary. Nauka, Novosibirsk.

1968

Group Properties of the Gas Dynamics Equations. Izd. Novosibirsk Univ., Novosibirsk.

On an analytic property of the gas dynamics equations and a theorem on the sonic plane associated with it. In *Third All-Union Congress on Theoretical and Applied Mechanics*, Moscow, 1968: Book of Abstracts. Nauka, Moscow, p. 233.

The perturbations of the unsteady motion of an incompressible liquid with a free boundary. Public address at a theoretical seminar at the Institute of Hydrodynamics of the Siberian Branch of the Academy of Sciences of the USSR. Izv. Akad. Nauk SSSR. MZhG, 3, 199.

Problems in Elementary Mathematics, Fifth Edition. Nauka, Moscow (together with V. B. Lidskii, A. N. Tulaikov and M. I. Shabunin).

Group properties of the gas dynamics equations. In *First Siberian Conference on Aerodynamics*. Book of Abstracts. Nauka, Novosibirsk, p. 10.

Group subdivision of boundary-layer equations. In *Continuum Dynamics. Inst. Gidrodinamiki, Sib. Otd. Akad. Nauk SSSR*, 1, pp. 24–25.

Partial invariance, Dokl. Akad. Nauk SSSR, **186**, 1, 22–25.

Nonstationary motions of incompressible fluid with free surfaces. In *Applied Mechanics: Proceedings of the 12th International Congress on Applied Mechanics*, Stanford, 1968. Springer, Berlin, p. 148.

On the disturbances of an unsteady motion of liquid with free boundary. In *Fluid Dynamics Transactions*. Warsaw, PWN, No. 4, pp. 105–113.

1969

Problems in Elementary Mathematics, Sixth Edition. Nauka, Moscow (together with V. B. Lidskii, A. N. Tulaikov and M. I. Shabunin).

1970

On the rising of a bubble. In *Some Problems of Mathematics and Mechanics*. Nauka, Leningrad, pp. 209–222.

1971

A One-dimensional Gas Dynamics Model. Inst. Gidrodinamiki, Sib. Otd. Akad. Nauk SSSR, Novosibirsk.

The group property of defining equations. In *Continuum Dynamics. Collection of Scientific Papers.* Inst Gidrodinamiki, Sib. Otd. Akad. Nauk SSSR, No. 7, pp. 5–11.

The plane problem of the unsteady motion of an incompressible liquid with a free boundary. Ibid, No. 8, pp. 22–26.

The non-linear Cauchy problem in the scale of Banach spaces. *Dokl. Akad. Nauk SSSR*, **200**, 4, 789–792.

Non-local Cauchy problems in fluid dynamics. In *Acts du Congr. Intern. des Mathématiciens*, Nice, France, 1970, Villars–Gauthier, Paris, Vol. 3, pp. 137–142.

1972

Analytic groups. *Introduction to the Theory of Infinite Continuous Groups of Transformations: Lectures.* Izd. Novisibirsk. Univ., Novosibirsk.

Non-local Cauchy problems in hydrodynamics. In *International Congress of Mathematicians*, Nice, 1970; Lectures of Soviet Mathematicians. Nauka, Moscow, pp. 225–230.

Group properties of the equations of mechanics. In *Continuum Mechanics and Related Problems of Analysis*. Nauka, Moscow, pp. 381–393.

A case of the unsteady motion of a liquid with a free boundary. In *Dynamics of a Continuum: Collection of Scientific Papers.* Inst Gidrodinamiki, Sib. Otd. Akad. Nauk SSSR, No. 12, pp. 124–130.

Approximate method for the scaling of the propagation law of one-dimensional shock waves. *Zh. Prikl. Mekh. Tekhn. Fiz.*, 1, 55–57.

1973

Problems in Elementary Mathematics, Seventh revised and supplemented Edition. Nauka, Moscow (together with V. B. Lidskii, A. N. Tulaikov and M. I. Shabunin).

The justification of shallow-water theory. In *Continuum Dynamics. Collection of Scientific Papers*. Inst Gidrodinamiki, Sib. Otd. Akad. Nauk SSSR, No. 15, pp. 104–125.

1974

Collection of Problems in Hydrodynamics. Izd. Novosibirsk. Univ., Novosibirsk (together with V. V. Pukhnachev, D. N. Gorelov, V. M. Men'shchikov and V. M. Teshukov).

Cauchy problems in the scale of Banach spaces of analytic functions. In *Proceedings of the Symposium on the Continuum Mechanics and Related Problems of Analysis*, Tbilisi, 1971. Metsniyereba, Tbilisi, Vol. 2, pp. 219–229.

To the shallow water theory foundation. *Arch. Mech.*, **26**, 3, 407–422.

1975

Group analysis of the differential equations of mechanics. In *Advances in Science and Technology*, General Mechanics. Vsesoyuz. Inst. Nauch. Tekh. Inform., Moscow, Vol. 2; pp. 5–52 (together with N. Kh. Ibragimov).

The differential invariants of a local Lie group of transformations. *Dokl. Akad. Nauk SSSR*, **222**, 6, 1291–1294.

Review: W. Noll *The Foundation of Mechanics and Thermodynamics*, Springer, Berlin, 1974. *Uspekhi Fiz. Nauk*, **117**, 2, 387–388.

1976

Introduction to Continuum Mechanics. A Student Text, Part 1. General Introduction. Izd. Novosibirsk. Univ., Novosibirsk.

The Cauchy–Poisson problem on a sphere. Vestnik Lenignrad Gos. Univ., *Matematika, Mekhanika*, **13**, 3, 146–153.

Cauchy problem in a scale of Banach spaces and its application to the shallow water theory justification. In *Lecture Notes in Mathematics*. Springer, Berlin, Vol. 503, pp. 426–437.

Group properties of differential equations. In *Mathematical and Numerical Methods in Fluid Dynamics*. Vienna, pp. 41–79.

Nonlinear and non-local Cauchy problem in hydrodynamics. Ibid, pp. 167–176.

Investigations on Cauchy–Poisson problems. In *Theoret. and Appl. Mech.* 14th IUTAM Congr., Delft, 1976; Book of Abstracts. Amsterdam, pp. 86–87.

The theorem of a flat sonic surface. In *Sympos. Transonicum II*, Göttingen, 1975. Springer, Berlin, pp. 165–173.

1977

Introduction to Continuum Mechanics. Student Textbook, Part 2, *Classical Models of the Mechanics of Continua*. Izd. Novosibirsk Univ., Novosibirsk.

A word of hydrodynamics. In *Continuum Dynamics. Collection of Scientific Papers*. Inst. Gidrodinamiki Sib. Otd. Akad. Nauk SSSR, No. 30, pp. 5–6.

1978

Group Analysis of Differential Equations. Nauka, Moscow.

The canonical structure of infinite Lie groups. In *Group-Theory Methods in Mechanics*. Proceedings of the International Symposium. Novosibirsk, pp. 188–201.

Foundation of shallow-water theory. In *Proceedings of the All-Union Conference on Partial Differential Equations Associated with the Seventieth Birthday of Academician I. G. Petrovskii*, Moscow, 1976. Izd. Moskovsk. Gos. Univ., Moscow, pp. 185–188.

Editor: *Group-Theory Methods in Mechanics*. Proceedings of the International Symposium, Novosibirsk (together with N. Kh. Ibragimov).

1979

Models of two-layer "shallow water". *Zh. Prikl. Mekh. Tekhn. Fiz.*, 2, 3–14.

1980

Abstract form of the Cauchy–Kovalevskii theorem and its applications. In *Partial Differential Equations*. Nauka, Novisibirsk, pp. 88–94.

1981

Lectures on the Foundations of Gas Dynamics. A Student Textbook. Nauka, Moscow.

1982

Group Analysis of Differential Equations. Academic Press, New York.

The quasidifferential operator in the Cauchy–Poisson problem. In *All-Union Conference "Lavrent'yev Lectures in Mathematics, Mechanics and Physics"*. Synopses of Lectures. Inst. Gidrodinamiki, Sib. Otd. Akad. Nauk SSSR, Novosibirsk, pp. 21–23.

1983

Cnoidal wave parameters. In *Problems of Mathematics and Mechanics: Collection of Scientific Papers in Memory of Academician M. A. Lavrent'yev*. Nauka, Novosibirsk, pp. 150–166.

1985

Wave Motions of Continua. Methodical Elaboration. Izd. Novisibirsk. Univ., Novosibirsk.

To the theory of non-linear waves. In *The Second All-Union Conference "Lavrent'yev Lectures in Mathematics, Mechanics and Physics"*. Synopses of Lectures. Inst. Matematiki, Akad. Nauk UkrSSR, Kiev, p. 94.

Lagrange approximations in the theory of waves. In *Non-linear Problems in the Theory of Surface and Internal Waves*. Nauka, Novosibirsk, pp. 10–77.

Non-linear Problems in the Theory of Surface and Internal Waves. Nauka, Novosibirsk (together with N. I. Makarenko, V. I. Nalimov, V. Yu. Lyapidevskii, P. I. Plotnikov, I. V. Sturova, V. I. Bukreyev and V. A. Vladimirov).

Editor: *Non-linear Problems in the Theory of Surface and Internal Waves*. Nauka, Novosibirsk (together with V. N. Monakhov).

1986

To the theory of non-linear waves. In *Annotated Lectures presented at the Sixth All-Union Congress on Theoretical and Applied Mechanics*. Tashkent, 1986. Izd. Fan, Tashkent, pp. 493–494.

1987

To the theory of non-linear waves. In *Mechanics and Scientific and Technical Progress*, Vol. 2, Fluid Mechanics. Nauka, Moscow, pp. 200–210.

The development of hydrodynamics in the Siberian branch of the Academy of Sciences of the USSR. *Zh. Prikl. Mekh. Tekh. Fiz.*, 4, 3–22 (together with B. A. Lugovtsov).

1990

Editor: M. A. Lavrent'yev, *Selected Papers. Mathematics and Mechanics.* Nauka, Moscow (together with N. N. Bogolyubov, V. S. Vladimirov *et al.*).

1991

On the asymptotic representation of solitary waves. *Dokl. Akad. Nauk SSSR*, **318,** 3, 556–559.

Exact results in the theory of waves on water. In *Proceedings of the All-Union Conference "Non-linear Phenomena"*. Nauka, Moscow, pp. 133–139.

1992

The PODMODELI Program. Inst. Gidrodinamiki, Sib. Otd. Ross. Akad. Nauk, Novosibirsk.

1993

On the property of x-autonomy. *Dokl. Ross. Akad. Nauk*, **330,** 5, 559–561.

On optimal systems of subalgebras. *Dokl. Ross. Akad. Nauk*, **333,** 6, 702–704.

The group analysis algorithms. In *Modern Group Analysis: Advances in Analytical and Computational Methods in Mathematical Physics*. Kluwer, Dordrecht, pp. 277–289.

The symmetry of isentropic ideal gas flows. In *Abstracts of the International Workshop on Advances in Analytical Methods in Aerodynamics*. Miedzyzdroje, Poland, 1993, p. 41.

On optimal systems of subalgebras. In *Proceedings of the XI Russian Colloquium "Modern Group Analysis and Problems of Mathematical Modelling"*, Samara, 1993. Izd. Samara Univ., Samara, 1993, pp. 9–10.

1994

The PODMODELI program. Gas dynamics. *Prikl. Mat. Mekh.,* **58,** 4, 30–55.

Isobaric gas motions. *Differents. Uraveniya*, **30,** 10, 1792–1799.

The group analysis purposes. *Lie Groups and their Applications*, **1,** 1, 193–202.

An optimal systems of subalgebras. *Lie Groups and their Applications*, **1,** 2, 18–26.

The singular vortex. In *Proceedings of the International. School-Seminar "Analytical Methods and Optimization of Processes of a Liquid and a Gas SAMGOP-94"*. Arzamas, 1994, p. 87.

1995

The singular vortex. *Zh. Prikl. Mekh. Tekh. Fiz.*, **36,** 3, 45–52.

Double sound waves. *Sib. Mat. Zh.* **36,** 3, 611–618.

Regular and irregular partially invariant solutions. *Dokl. Ross. Akad. Nauk*, **343,** 2, 156–159.

Regular partially invariant submodels of gas dynamics equations. *J. Nonlin. Math. Phys.*, **2,** 3–4, 236–246 (together with A. P. Chupakhin).

1996

Invariant integral conservation laws. *Dokl. Ross. Akad. Nauk*, **351,** 5, 599–602.

Regular partly invariant submodels of the gas dynamics equations. *Prikl. Mat. Mekh.*, **60,** 6, 990–999 (together with A. P. Chupakhin).

Regular type (2, 1) submodels of the gas dynamics equations. *Zh. Prikl. Mekh. Tekh. Fiz.*, **37,** 2, 3–13.

1997

Canonical form of invariant models of gas dynamics. Preprint No. 3–97. Inst. Gidrodinamiki, Sib. Otd. Ross. Akad. Nauk, Novosibirsk.

Partially invariant solutions to gas dynamics equations. In *Abstracts of the International Modern Group Analysis VII*. Lie Groups and Contemporary Symmetry Analysis, Sophus Lie Conference Center. Nordfjordeid, Norway, 1997, p. 49.

Development of the fluid mechanics at the Institute of Hydrodynamics of the Siberian Branch of the Russian Academy of Sciences during the years from 1986 to 1996. *Zh. Prikl. Mekh. Tekh. Fiz.*, **38,** 4, 3–27 (together with B. A. Lugovtsov).

1998

Plane flows of a gas with closed streamlines. *Dokl. Ross. Akad. Nauk*, **361,** 1, 51–53.
On the hierarchy of invariant submodels of differential equations. *Dokl. Ross. Akad. Nauk*, **361,** 6, 740–742.

1999

On "simple" solutions of equations of the dynamics of a polytropic gas. *Zh. Prikl. Mekh. Tekh. Fiz.*, **40,** 2, 5–12.
Some results of the implementation of the "PODMELI" program for the gas dynamics equations. *Prikl. Mat. Mekh.*, **63,** 3, 362–372.

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